In Figs. 5(e)–5(j), we can also see that the retrieval results from numerical simulations agree very well with the analytical results given by Eqs. (29). It verifies that the ω dependences of the optical parameters are valid in our previous work about realizing the repulsive Casimir force [36] using chiral metamaterials. Note that from Eq. (25), the strength of electric polarization and magnetic polarization can be modified independently by the length of the straight wires and the area of the circular loop respectively. $\Omega_\varepsilon > \Omega_\mu$ here is just one case. But no matter how to change the dimensional parameters, the relation Eq. (30) roughly always holds in such designs composited by the passive material only.

4. Conclusion

In summary, we have analyzed the wave propagation in chiral metamaterials and followed the similar procedure that was done in ordinary metamaterials to do the parameter retrieval for CMM slabs. Based on the transfer matrix technique, we extended the parameter retrieval technique to be able to treat samples with substrates and extra top layers. We studied the influence of the substrate and top layers on the thin chiral metamaterial slabs. We found that the substrate could lower the frequency of the resonance and induce the homogeneous chiral metamaterials to be inhomogeneous. The sensitivity of the influence relates to the strength of the leaked evanescent field in the substrate. We also fitted the retrieval results using analytical expressions derived from Ω-particle chiral model based on the effective LC circuit mode. We found they agree with each other very well.

A. Derive the forms of the constitutive parameters from the Ω-particle resonator model

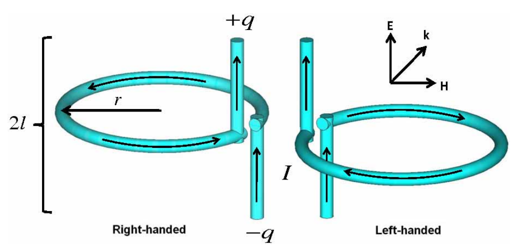

Fig. 6. Schematics of the single right-handed (left) and left-handed (right) Ω-particle resonators.

Figure 6 gives the schematics of the Ω-particle resonator. The structure consists of an open circular loop and two short wires. The area of the loop is $S = \pi r^2$ and the length of each wire is l. The wires are perpendicular to the loop and connected to the ends of the open loop. If the Ω-particle is in a homogeneous external field, the driving electric potential can be written as:

$$U = 2lE_0 \pm \mu_0 S\dot{H}_0, \tag{21}$$

where $\pm$ signs correspond to right-handed and left-handed helix resonators, The hat-dot denotes the first time derivative. Applying the effective RLC circuit model, we have

$$L\dot{I} + \frac{q}{C} + RI = U, \quad I = \dot{q}. \tag{22}$$

Let $\alpha = \frac{l}{L}$, $\beta = \mu_0 \frac{A}{L}$, $\gamma = \frac{R}{L}$, and $\omega_0^2 = \frac{1}{LC}$. Then assuming the external fields are plane waves, $\sim e^{-i\omega t}$, and considering stationary solutions, we get the following equation:

$$(-\omega^2 - i\omega\gamma + \omega_0^2)q = \alpha E_0 \pm i\omega\beta H_0, \tag{23}$$

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then, we have

$$q = \frac{\alpha}{-\omega^2 - i\omega\gamma + \omega_0^2} E_0 \pm \frac{i\omega\beta}{-\omega^2 - i\omega\gamma + \omega_0^2} H_0. \tag{24}$$

Therefore, the electric dipole, $\mathbf{p} = q\mathbf{l}$, and the magnetic dipole, $\mathbf{m} = \pm I\mathbf{A} = \pm\dot{q}\mathbf{A}$, can be written as:

$$\mathbf{p} = \frac{\alpha\mathbf{l}}{\omega_0^2 - \omega^2 - i\omega\gamma} E_0 + \frac{\pm i\omega\beta\mathbf{l}}{\omega_0^2 - \omega^2 - i\omega\gamma} H_0, \tag{25a}$$

$$\mathbf{m} = \frac{\mp i\omega\alpha\mathbf{A}}{\omega_0^2 - \omega^2 - i\omega\gamma} E_0 + \frac{\omega^2\beta\mathbf{A}}{\omega_0^2 - \omega^2 - i\omega\gamma} H_0, \tag{25b}$$

where, $\alpha\mathbf{A} = \frac{1}{\mu_0}\beta\mathbf{l}$, $\mathbf{l} = l\hat{\mathbf{l}}_0$, and $\mathbf{A} = A\hat{\mathbf{l}}_0$; $\hat{\mathbf{l}}_0$ is the unit vector along the direction of the wires, i.e., the up direction. The electric polarization and the magnetic polarization are defined as $\mathbf{P} = \frac{\sum\mathbf{p}}{V_0}$ and $\mathbf{M} = \frac{\sum\mathbf{m}}{V_0}$ respectively, where the summation is done in the unit cell and V_0 denotes the volume of the unit cell. Assuming there are N resonators in one unit cell, the electric polarization and the magnetic polarization can be written as:

$$\mathbf{P} = \frac{N}{V_0}\frac{\alpha l}{\omega_0^2 - \omega^2 - i\omega\gamma}\mathbf{E} + \frac{N}{V_0}\frac{\pm i\omega\beta l}{\omega_0^2 - \omega^2 - i\omega\gamma}\mathbf{H}, \tag{26a}$$

$$\mathbf{M} = \frac{N}{V_0}\frac{\mp i\omega\alpha A}{\omega_0^2 - \omega^2 - i\omega\gamma}\mathbf{E} + \frac{N}{V_0}\frac{\omega^2\beta A}{\omega_0^2 - \omega^2 - i\omega\gamma}\mathbf{H}. \tag{26b}$$

Here, the directions of $\mathbf{l}$ and $\mathbf{A}$ are merged into E_0 and H_0 to form the vectors of $\mathbf{E}$ and $\mathbf{H}$.

Inserting Eqs. (26) to the relation $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ and $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$, we have

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \frac{\alpha l N/V_0}{\omega_0^2 - \omega^2 - i\omega\gamma}\mathbf{E} + \frac{\pm i\omega\beta l N/V_0}{\omega_0^2 - \omega^2 - i\omega\gamma}\mathbf{H}, \tag{27a}$$

$$\mathbf{B} = \mu_0\mathbf{H} + \frac{\mp i\mu_0\omega\alpha AN/V_0}{\omega_0^2 - \omega^2 - i\omega\gamma}\mathbf{E} + \frac{\omega^2\mu_0\beta AN/V_0}{\omega_0^2 - \omega^2 - i\omega\gamma}\mathbf{H}. \tag{27b}$$

Comparing Eqs. (27) with Eq. (1), we can obtain the relative permittivity ε, permeability μ, and the chirality κ:

$$\varepsilon = 1 + \frac{\alpha l N/V_0\varepsilon_0}{\omega_0^2 - \omega^2 - i\omega\gamma}, \tag{28a}$$

$$\mu = 1 + \frac{\omega^2\beta AN/V_0}{\omega_0^2 - \omega^2 - i\omega\gamma}, \tag{28b}$$

$$\kappa = \frac{\pm\omega\beta l c_0 N/V_0}{\omega_0^2 - \omega^2 - i\omega\gamma} = \frac{\pm\omega\mu_0 c_0\alpha AN/V_0}{\omega_0^2 - \omega^2 - i\omega\gamma}. \tag{28c}$$

Equations (28) are valid for the case that the metal chiral structure stands in free space. If it's merged into a background material with ε_b and μ_b, Eqs. (28) should be modified as follows:

$$\varepsilon = \varepsilon_b + \frac{\Omega_\varepsilon\omega_0^2}{\omega_0^2 - \omega^2 - i\omega\gamma}, \tag{29a}$$

$$\mu = \mu_b + \frac{\Omega_\mu\omega^2}{\omega_0^2 - \omega^2 - i\omega\gamma}, \tag{29b}$$

$$\kappa = \frac{\Omega_\kappa\omega_0\omega}{\omega_0^2 - \omega^2 - i\omega\gamma}, \tag{29c}$$

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where ε_b is usually larger than one; μ_b is usually very close to one. Ω_ε, Ω_μ, and Ω_κ are the coefficients of the resonance terms in ε, μ, and κ, i.e. $\Omega_\varepsilon = \frac{\alpha l N}{V_0 \varepsilon_0 \omega_0^2}$, $\Omega_\mu = \frac{\beta A N}{V_0}$, and $\Omega_\kappa = \frac{\beta l c_0 N}{V_0 \omega_0} = \frac{\mu_0 c_0 \alpha A N}{V_0 \omega_0}$, where c_0 is the speed of light in vacuum. They describe the strength of the resonance. The expressions of the frequency dependence of ε and μ given by Eqs. (29a) and (29b) are the same as those in traditional metamaterials [37]. The linear ω dependence of κ is the same as Condon model for the homogeneous chiral molecular media [38]. This is a general feature of natural optically active materials [39].

Because $\alpha = \frac{l}{L}$ and $\beta = \mu_0 \frac{A}{L}$, we then can obtain the following relation:

$$\Omega_\kappa^2 = \Omega_\varepsilon \Omega_\mu, \tag{30}$$

which seems to limit the possibility of obtaining large Ω_κ in these designs composited by the passive materials only.

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