interior sheet is used for guiding the light back to the exterior layer. Fortunately, the Kepler profile (5) does not lead to total reflection if $r_0 \geq 2|w_2 - w_1|$. In this case, the invisible area is largest for

$$r_0 = 2|w_2 - w_1| \quad (7)$$

Figure 3 illustrates the light propagation in a dielectric invisibility device based on the simple map (3) and the Kepler profile (5) with $r_0 = 8a$. Here n ranges from 0 to about 36, but this example is probably not the optimal choice. One can choose from infinitely many conformal maps $w(z)$ that possess the required properties for achieving invisibility: $w(z) \sim z$ for $z \to \infty$ and two branch points w_1 and w_2. The invisible region may be deformed to any simply connected domain by a conformal map that is the numerical solution of a Riemann-Hilbert problem (*16*). We can also relax the tacit assumption that w_1 connects the exterior to only one interior sheet, but to m sheets where light rays return after m cycles. If we construct $w(z)$ as $af(z/a)$ with some analytic function $f(z)$ of the required properties and a constant length scale a, the refractive-index profile $|dw/dz|$ is identical for all scales a. Finding the most practical design is an engineering problem that depends on practical demands. This problem may also inspire further mathematical research on conformal maps in order to find the optimal design and to extend our approach to three dimensions.

Finally, we ask why our scheme does not violate the mathematical theorem (*3*) that perfect invisibility is unattainable. The answer is that waves are not only refracted at the boundary between the exterior and the interior layer, but also are reflected, and that the device causes a time delay. However, the reflection can be substantially reduced by making the transition between the layers gradual over a length scale much larger than the wavelength $2\pi/k$ or by using anti-reflection coatings. In this way, the imperfections of invisibility can be made as small as the accuracy limit of geometrical optics (*1*), i.e., exponentially small. One can never completely hide from waves, but can from rays.

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Controlling Electromagnetic Fields

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Using the freedom of design that metamaterials provide, we show how electromagnetic fields can be redirected at will and propose a design strategy. The conserved fields—electric displacement field D, magnetic induction field B, and Poynting vector B—are all displaced in a consistent manner. A simple illustration is given of the cloaking of a proscribed volume of space to exclude completely all electromagnetic fields. Our work has relevance to exotic lens design and to the cloaking of objects from electromagnetic fields.

To exploit electromagnetism, we use materials to control and direct the fields: a glass lens in a camera to produce an image, a metal cage to screen sensitive equipment, "blackbodies" of various forms to prevent unwanted reflections. With homogeneous materials, optical design is largely a matter of choosing the interface between two materials. For example, the lens of a camera is optimized by altering its shape so as to minimize geometrical aberrations. Electromagnetically inhomogeneous materials offer a different approach to control light; the introduction of specific gradients in the refractive index of a material can be used to form lenses and other optical elements, although the types and ranges of such gradients tend to be limited.

A new class of electromagnetic materials (*1*, *2*) is currently under study: metamaterials, which owe their properties to subwavelength details of structure rather than to their chemical composition, can be designed to have properties difficult or impossible to find in nature. We show how the design flexibility of metamaterials can be used to achieve new electromagnetic devices and how metamaterials enable a new paradigm for the design of electromagnetic structures at all frequencies from optical down to DC.

Progress in the design of metamaterials has been impressive. A negative index of refraction (*3*) is an example of a material property that does not exist in nature but has been enabled by using metamaterial concepts. As a result, negative refraction has been much studied in recent years (*4*), and realizations have been reported at both GHz and optical frequencies (*5*–*8*). Novel magnetic properties have also been reported over a wide spectrum of frequencies. Further information on the design and construction of metamaterials may be found in (*9*–*13*). In fact, it is now conceivable that a material can be constructed whose permittivity and permeability values may be designed to vary independently and arbitrarily throughout a material, taking positive or negative values as desired.

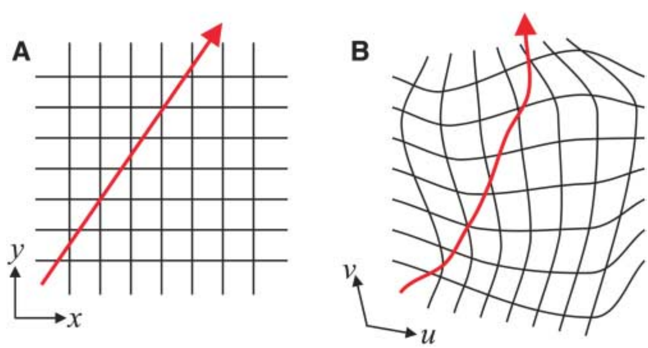

Fig. 1. (**A**) A field line in free space with the background Cartesian coordinate grid shown. (**B**) The distorted field line with the background coordinates distorted in the same fashion. The field in question may be the electric displacement or magnetic induction fields **D** or **B**, or the Poynting vector **S**, which is equivalent to a ray of light.

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If we take this unprecedented control over the material properties and form inhomogeneous composites, we enable a powerful form of electromagnetic design. As an example of this design methodology, we show how the conserved quantities of electromagnetism—the electric displacement field **D**, the magnetic field intensity **B**, and the Poynting vector **S**—can all be directed at will, given access to the appropriate metamaterials. In particular, these fields can be focused as required or made to avoid objects and flow around them like a fluid, returning undisturbed to their original trajectories. These conclusions follow from exact manipulations of Maxwell's equations and are not confined to a ray approximation. They encompass in principle all forms of electromagnetic phenomena on all length scales.

We start with an arbitrary configuration of sources embedded in an arbitrary dielectric and magnetic medium. This initial configuration would be chosen to have the same topology as the final result we seek. For example, we might start with a uniform electric field and require that the field lines be moved to avoid a given region. Next, imagine that the system is embedded in some elastic medium that can be pulled and stretched as we desire (Fig. 1). To keep track of distortions, we record the initial configuration of the fields on a Cartesian mesh, which is subsequently distorted by the same pulling and stretching process. The distortions can now be recorded as a coordinate transformation between the original Cartesian mesh and the distorted mesh

$$u(x,y,z), v(x,y,z), w(x,y,z) \tag{1}$$

where (u, v, w) is the location of the new point with respect to the x, y, and z axes. What happens to Maxwell's equations when we substitute the new coordinate system? The equations have exactly the same form in any coordinate system, but the refractive index—or more exactly the permittivity ε and permeability μ—are scaled by a common factor. In the new coordinate system, we must use renormalized values of the permittivity and permeability:

$$\varepsilon'_u = \varepsilon_u \frac{Q_u Q_v Q_w}{Q_u^2},$$
$$\mu'_u = \mu_u \frac{Q_u Q_v Q_w}{Q_u^2}, \text{ etc.} \tag{2}$$

$$E'_u = Q_u E_u,\ H'_u = Q_u H_u, \text{ etc.} \tag{3}$$

where,

$$Q_u^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2$$
$$Q_v^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2$$
$$Q_w^2 = \left(\frac{\partial x}{\partial w}\right)^2 + \left(\frac{\partial y}{\partial w}\right)^2 + \left(\frac{\partial z}{\partial w}\right)^2 \tag{4}$$

As usual,

$$\mathbf{B}' = \mu_0 \mu' \mathbf{H}', \quad \mathbf{D}' = \varepsilon_0 \varepsilon' \mathbf{E}' \tag{5}$$

We have assumed orthogonal coordinate systems for which the formulae are particularly simple. The general case is given in (*14*) and in the accompanying online material (*15*). The equivalence of coordinate transformations and changes to ε and μ has also been referred to in (*16*).

Now let us put these transformations to use. Suppose we wish to conceal an arbitrary object contained in a given volume of space; furthermore, we require that external observers be unaware that something has been hidden from them. Our plan is to achieve concealment by cloaking the object with a metamaterial whose function is to deflect the rays that would have struck the object, guide them around the object, and return them to their original trajectory.

Our assumptions imply that no radiation can get into the concealed volume, nor can any radiation get out. Any radiation attempting to penetrate the secure volume is smoothly guided around by the cloak to emerge traveling in the same direction as if it had passed through the empty volume of space. An observer concludes that the secure volume is empty, but we are free to hide an object in the secure space. An alternative scheme has been recently investigated for the concealment of objects (*17*), but it relies on a specific knowledge of the shape and the material properties of the object being hidden. The electromagnetic cloak and the object concealed thus form a composite whose scattering properties can be reduced in the lowest order approximation: If the object changes, the cloak must change, too. In the scheme described here, an arbitrary object may be hidden because it remains untouched by external radiation. The method leads, in principle, to a perfect electromagnetic shield, excluding both propagating waves and near-fields from the concealed region.

For simplicity, we choose the hidden object to be a sphere of radius R_1 and the cloaking region to be contained within the annulus $R_1 < r < R_2$. A simple transformation that achieves the desired result can be found by taking all fields in the region $r < R_2$ and compressing them into the region $R_1 < r < R_2$,

$$r' = R_1 + r(R_2 - R_1)/R_2,$$
$$\theta' = \theta,$$
$$\phi' = \phi \tag{6}$$

Applying the transformation rules (*15*) gives the following values: for $r < R_1$, ε' and μ' are free to take any value without restriction and do not contribute to electromagnetic scattering; for $R_1 < r < R_2$

$$\varepsilon'_{r'} = \mu'_{r'} = \frac{R_2}{R_2 - R_1}\frac{(r' - R_1)^2}{r'},$$
$$\varepsilon'_{\theta'} = \mu'_{\theta'} = \frac{R_2}{R_2 - R_1},$$
$$\varepsilon'_{\phi'} = \mu'_{\phi'} = \frac{R_2}{R_2 - R_1} \tag{7}$$

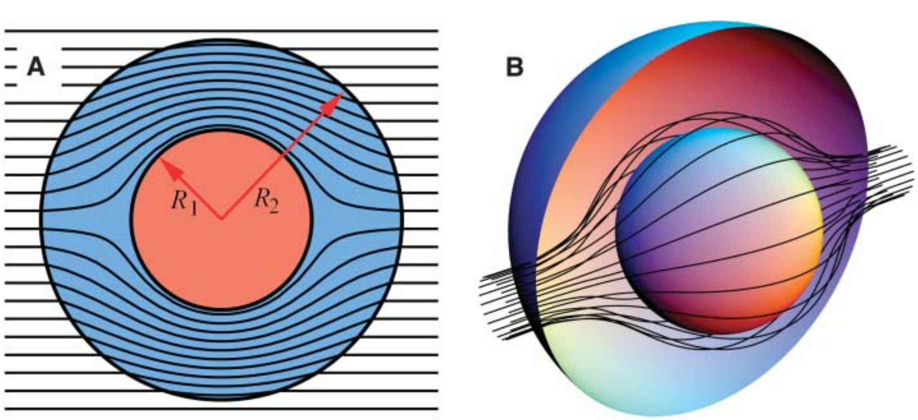

Fig. 2. A ray-tracing program has been used to calculate ray trajectories in the cloak, assuming that $R_2 \gg \lambda$. The rays essentially following the Poynting vector. (**A**) A two-dimensional (2D) cross section of rays striking our system, diverted within the annulus of cloaking material contained within $R_1 < r < R_2$ to emerge on the far side undeviated from their original course. (**B**) A 3D view of the same process.

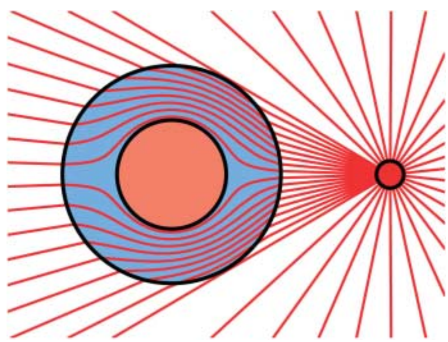

Fig. 3. A point charge located near the cloaked sphere. We assume that $R_2 \ll \lambda$, the near-field limit, and plot the electric displacement field. The field is excluded from the cloaked region, but emerges from the cloaking sphere undisturbed. We plot field lines closer together near the sphere to emphasize the screening effect.

for $r > R_2$

$$\varepsilon'_{r'} = \mu'_{r'} = \varepsilon'_{\theta'} = \mu'_{\theta'} = \varepsilon'_{\phi'} = \mu'_{\phi'} = 1 \quad (8)$$

We stress that this prescription will exclude all fields from the central region. Conversely, no fields may escape from this region. At the outer surface of the cloak ($r = R_2$), we have $\varepsilon'_{\theta'} = \varepsilon'_{\phi'} = 1/\varepsilon'_{r'}$ and $\mu'_{\theta'} = \mu'_{\phi'} = 1/\mu'_{r'}$, which are the conditions for a perfectly matched layer (PML). Thus we can make the connection between this cloak, which is reflectionless by construction, and a well-studied reflectionless interface (*18*).

For purposes of illustration, suppose that $R_2 \gg \lambda$, where λ is the wavelength, so that we can use the ray approximation to plot the Poynting vector. If our system is then exposed to a source of radiation at infinity, we can perform the ray-tracing exercise shown in Fig. 2. Rays in this figure result from numerical integration of a set of Hamilton's equations obtained by taking the geometric limit of Maxwell's equations with anisotropic, inhomogeneous media. This integration provides independent confirmation that the configuration specified by Eqs. 6 and 7 excludes rays from the interior region. Alternatively, if $R_2 \ll \lambda$ and we locate a point charge nearby, the electrostatic (or magnetostatic) approximation applies. A plot of the local electrostatic displacement field is shown in Fig. 3.

Next we discuss the characteristics of the cloaking material. There is an unavoidable singularity in the ray tracing, as can be seen by considering a ray headed directly toward the center of the sphere (Fig. 2). This ray does not know whether to be deviated up or down, left or right. Neighboring rays are bent around in tighter and tighter arcs the closer to the critical ray they are. This in turn implies very rapid changes in ε' and μ', as sensed by the ray. These rapid changes are due (in a self-consistent way) to the tight turn of the ray and the anisotropy of ε' and μ'. Anisotropy of the medium is necessary because we have compressed space anisotropically.

Although anisotropy and even continuous variation of the parameters is not a problem for metamaterials (*19–21*), achieving very large or very small values of ε' and μ' can be. In practice, cloaking will be imperfect to the degree that we fail to satisfy Eq. 7. However, very considerable reductions in the cross section of the object can be achieved.

A further issue is whether the cloaking effect is broadband or specific to a single frequency. In the example we have given, the effect is only achieved at one frequency. This can easily be seen from the ray picture (Fig. 2). Each of the rays intersecting the large sphere is required to follow a curved, and therefore longer, trajectory than it would have done in free space, and yet we are requiring the ray to arrive on the far side of the sphere with the same phase. This implies a phase velocity greater that the velocity of light in vacuum which violates no physical law. However, if we also require absence of dispersion, the group and phase velocities will be identical, and the group velocity can never exceed the velocity of light. Hence, in this instance the cloaking parameters must disperse with frequency and therefore can only be fully effective at a single frequency. We mention in passing that the group velocity may sometimes exceed the velocity of light (*22*) but only in the presence of strong dispersion. On the other hand, if the system is embedded in a medium having a large refractive index, dispersion may in principle be avoided and the cloaking operate over a broad bandwidth.

We have shown how electromagnetic fields can be dragged into almost any desired configuration. The distortion of the fields is represented as a coordinate transformation, which is then used to generate values of electrical permittivity and magnetic permeability ensuring that Maxwell's equations are still satisfied. The new concept of metamaterials is invoked, making realization of these designs a practical possibility.

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Nanoassembly of a Fractal Polymer: A Molecular "Sierpinski Hexagonal Gasket"

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Mathematics and art converge in the fractal forms that also abound in nature. We used molecular self-assembly to create a synthetic, nanometer-scale, Sierpinski hexagonal gasket. This non-dendritic, perfectly self-similar fractal macromolecule is composed of *bis*-terpyridine building blocks that are bound together by coordination to 36 Ru and 6 Fe ions to form a nearly planar array of increasingly larger hexagons around a hollow center.

Fractal constructs are based on the incorporation of identical motifs that repeat on differing size scales (*1*). Examples of fractal shapes in nature include clouds, trees, waves on a lake, the human circulatory system, and mountains, to mention but a few. The study of fractals has moved from the field of pure mathematics to descriptions of nature that, in turn, have inspired artistic design. More recently, chemists have incorporated the fractal form in molecular synthesis. Since 1985, molecular trees, which generally branch in a binary (*2*) or ternary (*3*) pattern, have been synthesized with increasing size and structural complexity. Beyond their aesthetics, these dendrimers and hyperbranched materials (*4*) are now under study for use in a wide range of practical applications. However, treelike patterns are but one type of fractal composed of repeating geometrical figures. A porphyrin-based dendrimer (*5*) that uses porphyrins as branching centers has been prepared that incorporates the snakelike "kolam" fractal pattern described by Ascher (*6*).