Losses in left-handed materials

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Abstract: Interest in negative refractive index, or left-handed (LH) materials, has escalated rapidly over the last few years and it now appears that useful LH materials may be realizable in the microwave region. However there is also considerable interest in LH materials for infrared and visible applications. The purpose of this paper is to explore the limitations of LH materials at short wavelengths due to inherent losses. Our conclusions are that it may be quite difficult to achieve useful LH materials at wavelengths less than about 10 microns using current approaches.

OCIS codes: (160.4670) Optical Materials

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1. Introduction

Since the publication by Veselago [1] in 1968 indicating that materials with simultaneously negative permittivity and permeability should exhibit interesting and unusual properties there has been a rapidly increasing interest in the properties and possible applications of various metamaterials with these properties, often referred to as left handed or negative index materials [2-25]. In addition, Optics Express has included thirteen articles on Negative Refraction and Metamaterials in a special Focus Issue published on 7 April 2003. Most of the reported efforts, so far, have focused on RF or microwave applications. However, there has always been interest in determining if such materials with negative refractive index could be utilized at infrared and visible wavelengths (see, for example [6, 16, 20, and 24]). From the recent work of O'Brien and Pendry [16] it seems clear that it will be difficult to realize useful left handed materials at near infrared and shorter wavelengths using current approaches. Besides the increasing difficulty in actually making structures with regions of negative refractive index as the wavelength decreases, inherent absorption losses become a more formidable problem. The purpose of this paper is to examine this latter limitation more fully.

2. Formulation

The framework for all of this activity was set by Veselago in 1968 [1]. The constitutive equations for homogeneous isotropic materials are:

$$n^2 = \varepsilon\mu \tag{1}$$

$$k^2 = \frac{\omega^2}{c^2} n^2 \tag{2}$$

where n is the complex index of refraction, ε is the relative complex permittivity, μ is the relative complex permeability, and k is the complex propagation constant. Breaking these into real and imaginary parts we can obtain

$$n_2 = +\left\{\frac{1}{2}(\varepsilon_2\mu_2 - \varepsilon_1\mu_1) + \frac{1}{2}((\varepsilon_1^2 + \varepsilon_2^2)\cdot(\mu_1^2 + \mu_2^2))^{1/2}\right\}^{1/2} \tag{3}$$

and

$$n_1 = \frac{1}{2n_2}(\varepsilon_1\mu_2 + \mu_1\varepsilon_2)\,. \tag{4}$$

The subscripts 1, and 2 indicate real and imaginary parts respectively and ε_2, μ_2, n_2, k_1 and k_2 are positive. Note that from Eq. (4), the sign of n_1 depends on the sign of the quantity $(\varepsilon_1\mu_2+\mu_1\varepsilon_2)$. The absorption per unit length in the material is

$$\Lambda_m = 2k_2 = 2\frac{\omega}{c}n_2 . \tag{5}$$

The wavelength in the material is

$$\lambda_m = \frac{2\pi}{|n_1|}\cdot\frac{c}{\omega} , \tag{6}$$

and the transmission through a single wavelength in the material is $\exp(-L_m)$ where

$$L_m = \Lambda_m \cdot \lambda_m = 4\pi\frac{n_2}{|n_1|} = 4\pi\frac{|\varepsilon_1\mu_2 + \mu_1\varepsilon_2|}{(\varepsilon_1\mu_1 - \varepsilon_2\mu_2) + ((\varepsilon_1^2 + \varepsilon_2^2)\cdot(\mu_1^2 + \mu_2^2))^{1/2}} . \tag{7}$$

is the material loss factor. The loss factor, in the material, per wavelength (in air) is

$$L_a = 4\pi n_2 \tag{8}$$

It will be seen below that these loss factors become particularly important when one considers applications of left-handed materials at short wavelengths. However, losses can also be quite large even in the microwave region, as we will see below.

3. Applications

Experimental verification of negative refractive index was first reported by Shelby, Smith and Schultz [9] at microwave frequencies. Their results indicate that a negative index of refraction was obtained for their structure between 10.2 and 10.8 GHz. They presented refracted power at 10.5 GHz from which they deduced an effective negative index of refraction of –2.7 ± 0.1. Using their model parameters, and the above equations we can calculate the expected loss for f = 10.5 GHz. We obtain $\Lambda_m = 0.40$ cm^{-1}, $\lambda_m = 0.78$ cm, L_m = 0.31. This indicates that about a third of the energy is lost per cm through the structure. This magnitude of loss would have to be considered in any application of left handed materials. In reference [9] the wedge used is about 3.1 cm thicker on one side than the other so the attenuation of radiation passing through the thick side will be about 5 dB greater than through the thin side. More recently Parazzoli *et al*. [22] have examined a somewhat modified structure at 12.6 GHz. Their calculated effective permittivities and permeabilities for their structure at 12.6 GHz are extremely low indicating very little expected loss in their structure.

Unfortunately the situation becomes more problematical at shorter wavelengths as the structure dimensions get smaller. As shown by O'Brien and Pendry [16], the dimensions of the structures can become comparable with the skin depth so both resistive losses and skin depth issues become problems at short wavelengths. O'Brein and Pendry have calculated the permittivity and permeability of several split-ring type structures with resonances in the infrared. Using the above equations and the parameters from Table 2 for the second split-ring structure of their paper we calculate the values of L_a and of L_m for their three structures shown in Table 1 below. Also shown is the resonance wavelength, λ'_o; the wavelength, in air, λ_a, at which L_a is a minimum; and the wavelength, in air, λ_{am}, at which L_m is a minimum for each structure.

Table 1 Loss parameters calculated for the second split ring structure considered in reference [16] using their parameters and analysis and the above equations. Rows are labeled by the unit cell dimensions (in nm) for the three structures considered.

	λ'_o(μm)	L_a	λ_a(μm)	L_m	λ_{am}(μm)
600	3.70	2.39	3.45	2.66	3.53
300	2.32	7.42	2.23	5.30	2.26
150	1.78	24.50	1.74	14.83	1.76

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These results indicate that in all three cases the wave propagation will be heavily damped. This is true even though ε_1 and n_1 are negative for all three structures and μ_1 is negative for the first two.

4. Analysis

The basic reason these losses are large is that μ_1 is negative only in the close vicinity of the resonance of the split-ring structures where μ_2 is large. To get some insight into this we will make some approximations. First, for all three structures $|\varepsilon_1| >> \varepsilon_2$. In this limit Eq. (7) becomes

$$L_m \cong 4\pi \frac{x}{a + (1 + x^2)^{1/2}} \tag{9}$$

where $a = \varepsilon_1\mu_1 / |\varepsilon_1\mu_1| = \pm 1$, and $x = \mu_2 / |\mu_1|$. The loss per wavelength in the material becomes independent of the permittivity although, of course, the wavelength itself does not. In Eq. (9) the minimum of L_m occurs at the minimum of x. From O'Brien and Pendry [16] the minimum of x occurs at the frequency ω where

$$\omega^2 - \omega'^2_o = \frac{1}{4}\Gamma^2 + \frac{1}{2} f' \omega'^2_o \tag{10}$$

Γ is related to the absorptive losses, f' is the effective fill factor for the split ring and ω_o' is the resonance frequency. In the range considered $\Gamma^2 << 2 f' \omega'^2_o$ so Eq. (10) becomes

$$\omega^2 \cong \omega'^2_o \left(1 + \frac{1}{4} f'\right) \tag{11}$$

Using their equations again we obtain

$\mu_1 = 1 - 2\dfrac{1}{1+y^2}$, and $\mu_2 = 2\dfrac{y}{1+y^2}$, where $y = \dfrac{2\Gamma}{f' \omega'_o}$, and

$$L_m = 4\pi y = 8\pi \frac{\Gamma}{f' \omega'_o} . \tag{12}$$

Using these approximations and the formulas and values of the parameters given in [16] we reproduce the results given in Table 1 for L_m and λ_{am} to within about $\pm 4\%$.

We can rewrite the Eq. (12) in terms of the dimensions of the second split-ring structure employed by O'Brien and Pendry [16] and the electromagnetic skin depth δ, where

$$\delta = \left(\frac{2\rho}{\mu_o \omega}\right)^{1/2} \tag{13}$$

ρ is the resistivity of the metal, μ_o is the permeability of free space, and ω is the angular frequency of the radiation. The result is

$$L_m = 8\pi \frac{\delta^2}{fdR} \tag{14}$$

where $f = \pi R^2 / a^2$, is the fill factor, R is the radius, and d is the width of the split ring. Equation (13) can be rewritten as $\delta = (\rho\lambda / \pi R_o)^{1/2}$ where λ is the free space wavelength of the radiation and R_o = 377Ω is the impedance of free space. Equation (14) indicates that losses will become very large as d and R become comparable to the skin depth. Equation (14) can be rewritten as

$$L_m = \frac{8\rho\lambda}{\pi R_o} \cdot \frac{a^2}{R^3 d} \tag{15}$$

In the structure discussed in [16] R and d are constrained by the equation $\pi(R+d)^2 = b^2$ where b is limited by the unit cell dimension, a, which, in turn, must be much less than the free-space wavelength. We can minimize Eq. (15) subject to this restriction. The result is $R = 3d = 3b/4\sqrt{\pi}$. Inserting this into Eq. (15)

$$L_m = 8\pi \frac{4^4}{3^3} \frac{\rho}{R_o} \cdot \left(\frac{a}{b}\right)^4 \cdot \frac{\lambda}{a^2} \tag{16}$$

Furthermore, according to O'Brien and Pendry [15] the lattice dimension, a, should be considerably smaller than λ so that the structure can be modeled as an "effectively homogeneous medium". We have taken λ_o' = 6a and in our calculations. For these the minimum value of L_m occurs at $\lambda \approx 0.97\lambda'_o \approx 5.8a$. In reference [16] b = 0.52a. For silver ρ = 1.62×10^{-8} Ωm and Eq. (16) becomes

$$L_m \cong \frac{4.8}{\lambda} \tag{17}$$

with λ measured in microns. This simple result indicates that it should be very difficult to achieve usable negative index, left-handed, materials at wavelengths shorter than mid-infrared by this method. As a final calculation we can numerically examine the expected propagation properties of optimized structures employing λ_o' = 6a. This scaling of λ_o' with lattice dimension can, in principle at least, be accomplished by adjusting the capacitance of the split ring. The results are shown in Fig. 1.

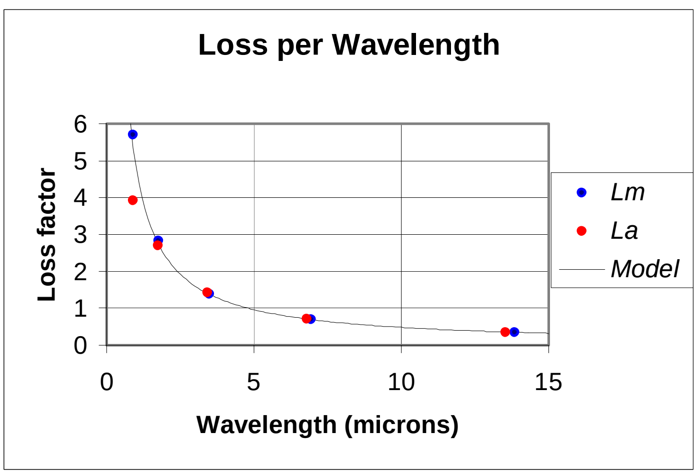

Fig. 1. Calculated loss factors in the modified structure are shown for the minimum loss per wavelength in the material, L_m, and the minimum loss per free-space wavelength, L_a, along with the model with L = 4.8/λ where λ is the corresponding wavelength in free space in microns.

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It is impressive that the simple model represented by Eq. (17) fits the calculated values of L_{m} this well . It is surprising, however, that L_a also fits the approximation as well as it does. Notice that the values of L_a and L_{m} calculated for the optimized structures are considerably less than those calculated for the initial structures of O'Brien and Pendry, given in Table 1. In addition for all of the optimized structures, even for $\lambda_o' < 1\mu\mathrm{m}$, μ_1 and ε_1 are both negative as is n_1 indicating that all the structures should result in left-handed propagation although the propagation is severely damped for the shorter wavelength structures. These results indicate that it might be possible to realize a useful LHM structure using this split-ring approach at wavelengths in the mid-infrared if one is willing to accept some modest losses. Much shorter wavelength structures appear likely to be too lossy to be useful.

5. Conclusions

The details of this analysis are, of course, specific to the split-ring structure discussed by O'Brien and Pendry [16]. Nevertheless, this is probably a fairly fundamental limit to the use of any split-ring resonator configuration to create negative index materials at short wavelengths. In addition, considerations of this type should be applicable to any negative index material particularly those in which the negative index occurs due to, and in the region of, an electromagnetic resonance as n_2 will probably not be negligible. Recently several authors [18, 20, and 24] have examined other types of structures to achieve negative index behavior at infrared and visible wavelengths. Panina *et al.* [18] have considered an optomagnetic composite medium with split-ring and antenna conducting microelements. It would appear that their structure should be subject to the same type of limitations as that of O'Brien and Pendry [16]. In addition, their results indicate that the imaginary parts of the electric and magnetic polarizability as well as the permittivity and permeability are quite large and that a negative index is difficult to achieve using their approach leading one to suspect that losses in their structure would be very large. Shvets [20] has proposed a photonic approach to making a material with a negative index of refraction using two dielectric materials, with positive and negative dielectric permittivities. In his analysis Shvets ignored the "small damping constant". However, a simplistic analysis indicates that L_{m} may not be negligible for his structure and a more complete analysis should be done. Podolskiy [24] has examined plasmon modes and negative refraction in metal nanowire composites. Again, no detailed analysis of expected losses was included. Our only intent in this paper is to indicate that losses in metamaterials designed to function as left-handed materials in the infrared and visible spectral regions are likely to be quite large and should be carefully included in the analysis of any proposed approach.