Reversing Light: Negative Refraction

First hypothesized in 1967 by Victor Veselago, materials with negative refractive index have only been appreciated this decade.

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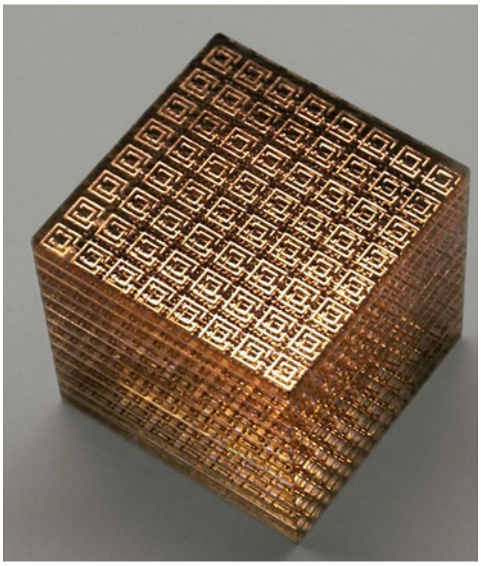

'The Boeing cube': a structure designed for negative refractive index in the GHz range

Victor Veselago, in a paper[1] published in 1967, pondered the consequences for electromagnetic waves interacting with a hypothetical material for which both the electric permittivity, ε, and the magnetic permeability, μ, were simultaneously negative. As no naturally occurring material or compound has ever been demonstrated with negative ε and μ, Veselago wondered whether this apparent asymmetry in material properties was just happenstance, or perhaps had a more fundamental origin. Veselago concluded that not only should such materials be possible but, if ever found, would exhibit remarkable properties unlike those of any known materials, giving a twist to virtually all electromagnetic phenomena.

So why are there no materials with negative ε and μ? We first need to understand what it means to have a negative ε or μ, and how it occurs in materials. The Drude-Lorentz model of a material is a good starting point, as it conceptually replaces the atoms and molecules of a real material by a set of harmonically bound electron oscillators, resonant at some frequency ω_0. At frequencies far below ω_0, an applied electric field displaces the electrons from the positive core, inducing a polarization in the same direction as the applied electric field. At frequencies near the resonance, the induced polarization becomes very large, as is typically the case in resonance phenomena; the large response represents accumulation of energy over many cycles, such that a considerable amount of energy is stored in the resonator (medium) relative to the driving field. So large is this stored energy that even changing the sign of the applied electric field has little effect on the polarization near resonance! That is, as the frequency of the driving electric field is swept through the resonance, the polarization flips from in-phase to out-of-phase with the driving field and the material exhibits a negative response. If instead of electrons the material response were due to harmonically bound magnetic moments, then a negative magnetic response would exist.

Though somewhat less common than positive materials, negative materials are nevertheless easy to find. Materials with ε negative include metals (e.g., silver, gold, aluminum) at optical frequencies, while materials with μ negative include resonant ferromagnetic or antiferromagnetic systems.

That negative material parameters occur near a resonance has two important consequences. First, negative material parameters will exhibit frequency dispersion: that is to say they will vary as a function of frequency. Second, the usable bandwidth of negative materials will be relatively narrow compared with positive materials. This can help us answer our initial question as to why materials with both negative ε and μ are not readily found. The resonances in existing materials that give rise to electric polarizations typically occur at very high frequencies, in the optical, for metals, or at least in the THz to infrared region for semiconductors and insulators. On the other hand, resonances in magnetic systems typically occur at much lower frequencies, usually tailing off toward the THz and infrared region. In short, the fundamental electronic and magnetic processes that give rise to resonant phenomena in materials simply do not occur at the same frequencies, although no physical law would preclude this.

Metamaterials extend material response

Because of the seeming separation in frequency between electric and magnetic resonant phenomena, Veselago's analysis of materials with ε and μ both negative might have remained a curious exercise in electromagnetic theory. However, in the mid-1990s, researchers began looking into the possibility of engineering artificial materials to have tailored electromagnetic response. While the field of artificial materials dates back to the 40s, advances in fabrication and computation—coupled with the emerging awareness of the importance of negative

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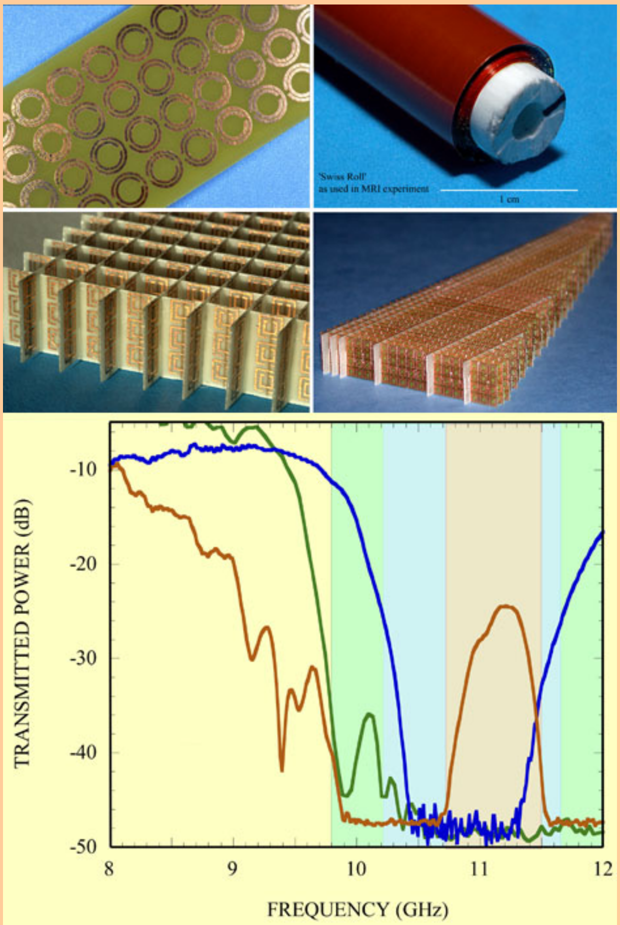

Figure 1. Transmission experiments resolve whether a material is transparent or opaque, providing an indication of the sign of the material parameter. Top panels: examples of metamaterials used in microwave experiments. The unit cells are of the order of 5mm across. Bottom panel: The transmitted power spectra[3] shown here are for (green) a metamaterial of cut wires; (blue) a metamaterial of split ring resonators (SRRs); and (red) a metamaterial of wires and SRRs combined. For the wires, the region of negative ε occurs at frequencies starting just below 10 GHz, while for SRRs the region of negative μ spans a 1 GHz region starting at ~10.3 GHz. The combined medium exhibits a transmission band that would lie in an opaque frequency region for either metamaterial alone.

materials—led to a resurgence of effort in developing new structures with novel material properties.

To form an artificial material, we start with a collection of repeated elements designed to have a strong response to applied electromagnetic fields. So long as the size and spacing of the elements are much smaller than the electromagnetic wavelengths of interest, incident radiation cannot distinguish the collection of elements from a homogeneous material. We can thus conceptually replace the inhomogeneous composite by a continuous material described by material parameters ε and μ. At lower frequencies, conductors are excellent candidates from which to form artificial materials, as their response to electromagnetic fields is large.

A metamaterial mimicking the Drude-Lorentz model can be straightforwardly achieved by an array of wire elements into which cuts are periodically introduced. The effective permittivity for the cut-wire medium, then, has the form,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

where the plasma frequency, ω_p, and the resonance frequency, ω_0, are determined only by the geometry of the lattice rather than by the charge, effective mass and density of electrons, as is the case in naturally occurring materials. At frequencies above ω_0 and below ω_p, the permittivity is negative and, because the resonant frequency can be set to virtually any value in a metamaterial, phenomena usually associated with optical frequencies—including negative ε—can be reproduced at low frequencies.

The path to achieving magnetic response from conductors is slightly different. From the basic definition of a magnetic dipole moment,

$$\mathbf{m} = \tfrac{1}{2}\int_V \mathbf{r} \times \mathbf{j}\, dV$$

we see that a magnetic response can be obtained if local currents can be induced to circulate in closed loops (solenoidal currents). Moreover, introducing a resonance into the element should enable a very strong magnetic response, potentially one that can lead to a negative μ.

In 1999, Pendry *et al*[2]. proposed a variety of structures that, they predicted, would form magnetic metamaterials. These structures consisted of loops or tubes of conductor with a gap inserted. From a circuit point of view, a time varying magnetic field induces an electromotive force in the plane of the element, driving currents within the conductor. A gap in the plane of the structure introduces capacitance into the planar circuit, giving rise to a resonance at a frequency set by the geometry of the element. This *split ring resonator* (SRR), in its various forms, can be viewed as the metamaterial equivalent of a magnetic atom. Pendry *et al.* went on to show that the SRR medium could be described by the resonant form

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

The wire medium and the SRR medium represent the two basic building blocks—one electric the other magnetic—for a large range of metamaterial response, including Veselago's hypothesized material (see figure 1).

Negative refraction

Maxwell's equations determine how electromagnetic waves propagate within a medium and can be solved to arrive at a wave equation of the form,

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \varepsilon\mu \frac{\partial^2 E(x,t)}{\partial t^2}$$

In this equation ε and μ enter as a product and it would not appear to matter whether the signs of ε and μ were both positive or were both negative. Indeed, solutions of the wave equation have the form $\exp\left[i(nkd - \omega t)\right]$ where $n = \sqrt{\varepsilon\mu}$ is the refractive index. Propagating solutions exist in the material whether ε and μ are both positive or are both negative. So what, if anything, is the difference between positive and negative materials?

It turns out that we need to be more careful in taking the square root, as ε and μ are analytic functions that are generally complex valued. There is an ambiguity in the sign of the square root that is resolved by a proper analysis. For example, if instead of writing $\varepsilon = -1$ and $\mu = -1$ we write $\varepsilon = \exp(i\pi)$ and $\mu = \exp(i\pi)$, then:

$$n = \sqrt{\varepsilon\mu} = \exp(i\pi/2)\exp(i\pi/2) = \exp(i\pi) = -1 .$$

The important step is that the square root of either ε or μ alone must have a positive imaginary part—this is necessary for a passive material. This briefly stated argument shows why the material Veselago pondered years ago is so unique: the index of refraction is negative.

A negative refractive index implies that the phase of a wave advancing through the medium will be negative rather than positive. As Veselago pointed out, this fundamental reversal of wave propagation contains important implications for nearly all electromagnetic phenomena. Many of the exotic effects of negative index have been or are currently being pursued by researchers. But perhaps the most immediately accessible phenomenon from an experimental or computational point-of-view is the reversal of wave refraction, illustrated in figure 2.

Snell's law, which describes quantitatively the bending of a wave as it enters a medium, is perhaps one of the oldest and most well known of electromagnetic phenomena. In the form of a wedge experiment, as depicted in figure 2, Snell's law is also the basis for a direct measurement of a material's refractive index. In this type of experiment, a wave is incident on the flat side of a wedge shaped sample. The wave is transmitted through the transparent sample, striking the second interface at an angle. Because of the difference in refractive index between the material and free space, the beam exits the wedge deflected by some angle from the direction of incidence.

One might imagine that an experimental determination of Snell's law should be a simple matter; however, the peculiarities of metamaterials add a layer of complexity that renders the experimental confirmation somewhat more difficult. Present samples, based on SRRs and wires, are frequency dispersive with fairly narrow bandwidths and exhibit considerable loss. The first experiment showing negative refraction was performed in 2001 by Shelby et al. at UCSD[4], who utilized a planar waveguide apparatus to reduce the dimensionality of the measurement to two dimensions, similar to that depicted in figure 2. Shelby et al. measured the power refracted from a two-dimensional wedge metamaterial sample as a function of angle, confirming the expected properties.

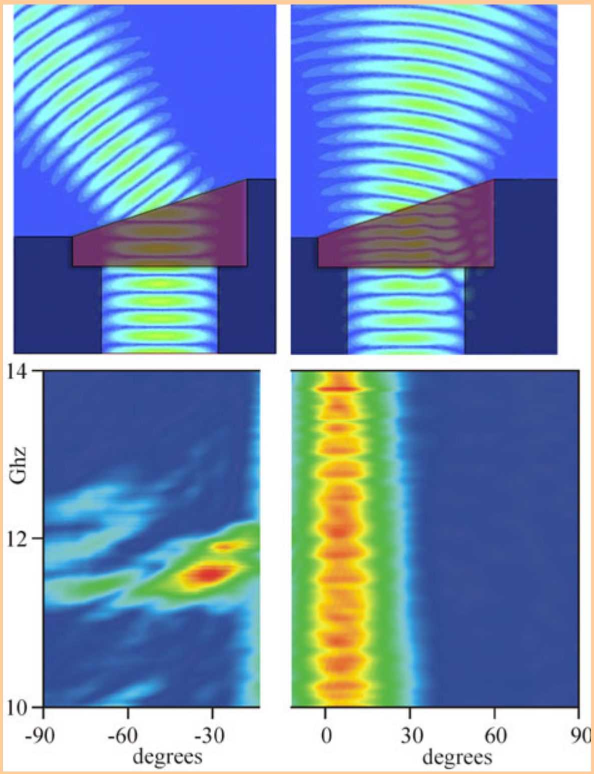

Figure 2. A negative index implies negative refraction. *Top left:* in this simulation[5] of a Snell's law experiment, a negative index wedge $\varepsilon = -1.0, \mu = -1.0$ deflects an electromagnetic beam, incident at an angle to the interface, by a negative angle so that it emerges on the same side of the surface normal as the incident beam, confirming negative refraction. *Top right:* by contrast a wedge with positive index $\varepsilon = 2.0, \mu = 1.0$ will positively refract the same beam relative to the surface normal. Experiments also confirm this behavior. *Bottom left:* deflection observed for a negative wedge. *Bottom right:* deflection (horizontal axis) observed for a Teflon wedge as a function of frequency (vertical axis). Note that in the latter case there is strong dispersion with frequency because the condition $\varepsilon = -1.0, \mu = -1.0$ is realized only over a narrow bandwidth around 12GHz.

While the UCSD data were compelling, the concept of negative index proved counter-intuitive enough that many other researchers needed further convincing. In 2003, Houck et al.[6] at MIT repeated the negative refraction experiment on the same sort of negative index metamaterial, confirming the original findings. The MIT group considered wedges with different angles, showing that the observed angle of refraction was consistent with Snell's law for the metamaterial. In the same year, Parazzoli et al.[7], from Boeing Phantom Works, also presented data on a negative index sample. The Boeing data differed from the previous two experiments in that their sample was designed not for a two-dimensional scattering chamber, but rather for free space. The Boeing experiment removed any doubts that the observed deflection might be due to any artifact related to the planar waveguide; moreover, the distance measured from the sample was significantly larger than for the previous waveguide demonstrations.

While it has proven a valuable concept, a rigorously defined negative index-of-refraction may not necessarily be a prerequisite for negative refraction phenomena. An alternate approach to attaining negative refraction uses the properties of 'photonic crystals'[8,9,10,11,12] - materials that lie on the transition between a metamaterial and an ordinary structured dielectric. Photonic crystals derive their properties from Bragg reflection in a periodic structure engineered in the body of a dielectric, typically by drilling or etching holes. The periodicity in photonic crystals is on the order of the wavelength, so that the distinction between refraction and diffraction is blurred. Nevertheless, many novel dispersion relationships can be realized in photonic crystals, including ranges where the frequency disperses negatively with wave vector as required for a negative refraction.

The concept of negative refraction has also been generalized to transmission line structures, common in electrical engineering applications. By pursuing the analogy between lumped circuit elements and material parameters, Eleftheriades and coworkers[13] have demonstrated negative refraction phenomena in microwave circuits. The transmission line model has proven exceptionally valuable for the development of microwave devices: Tatsuo Itoh and Christophe Caloz at UCLA have applied the transmission line model to develop novel microwave components, including antennas, couplers and resonators.

These experiments and applications have shown that the material Veselago hypothesized more than thirty-five years ago can now be realized using artificially constructed metamaterials, making discussion of negative refractive index more than a theoretical curiosity. The question of whether such a material can exist has been answered, turning the development of negative index structures into a topic of materials—or metamaterials—physics. As metamaterials are being designed and improved, we are now free to consider the ramifications associated with a negative index-of-refraction. This material property, perhaps because it is so simply stated, has enabled the rapid design of new electromagnetic structures—some of them with very unusual and exotic properties.

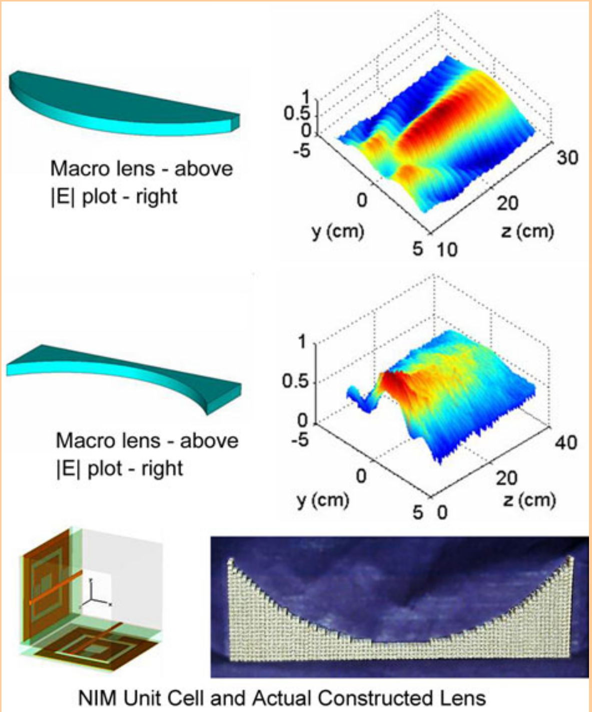

Figure 3. A material with index of $n = +1$ (relative to vacuum) has no refractive power, while a material with index of $n = -1$ has considerable refractive power. Consider the formula for the focal length of a thin lens: $f = R/|n-1|$, where R is the radius-of-curvature of the surface. For a given R, a lens with an index of $n = -1$ will have the same focal length as would an index of $n = +3$; by the same reasoning, if we compare two lenses of the same absolute value of index but opposite sign, the negative index lens will be more compact than the positive index lens. The practicality of negative index metamaterial lenses has been demonstrated by researchers at Boeing, Phantom Works. Applying the same basic elements previously used to construct negative index metamaterial wedges—SRRs to create a magnetic response, wires to create an electric response (see inset above)—Parazzoli, Greegor and their colleagues[14] have designed a concave lens with an index very near to $n = -1$ at microwave frequencies (~15 GHz). The negative index metalens has a much shorter focal length as compared with a positive index lens (in this case $n = 2.3$) having the same radius-of-curvature, as shown in the figures above. Moreover, the metalens is comparatively much lighter than the positive index lenses, a significant advantage for aerospace applications.

A better focus with negative index

Refraction is the phenomenon responsible for lenses and similar devices that focus or shape radiation. While usually thought of in the context of visible light, lenses are utilized throughout the electromagnetic spectrum, and represent a good starting point to implement negative index materials.

In his early paper, Veselago noted that a negative index focusing lens would need to be concave rather than convex. This would seem to be a trivial matter, but there is, in fact, more to the story. For thin lenses, geometrical optics—valid for either positive or negative index—gives the result that the focal length is related to the radius of

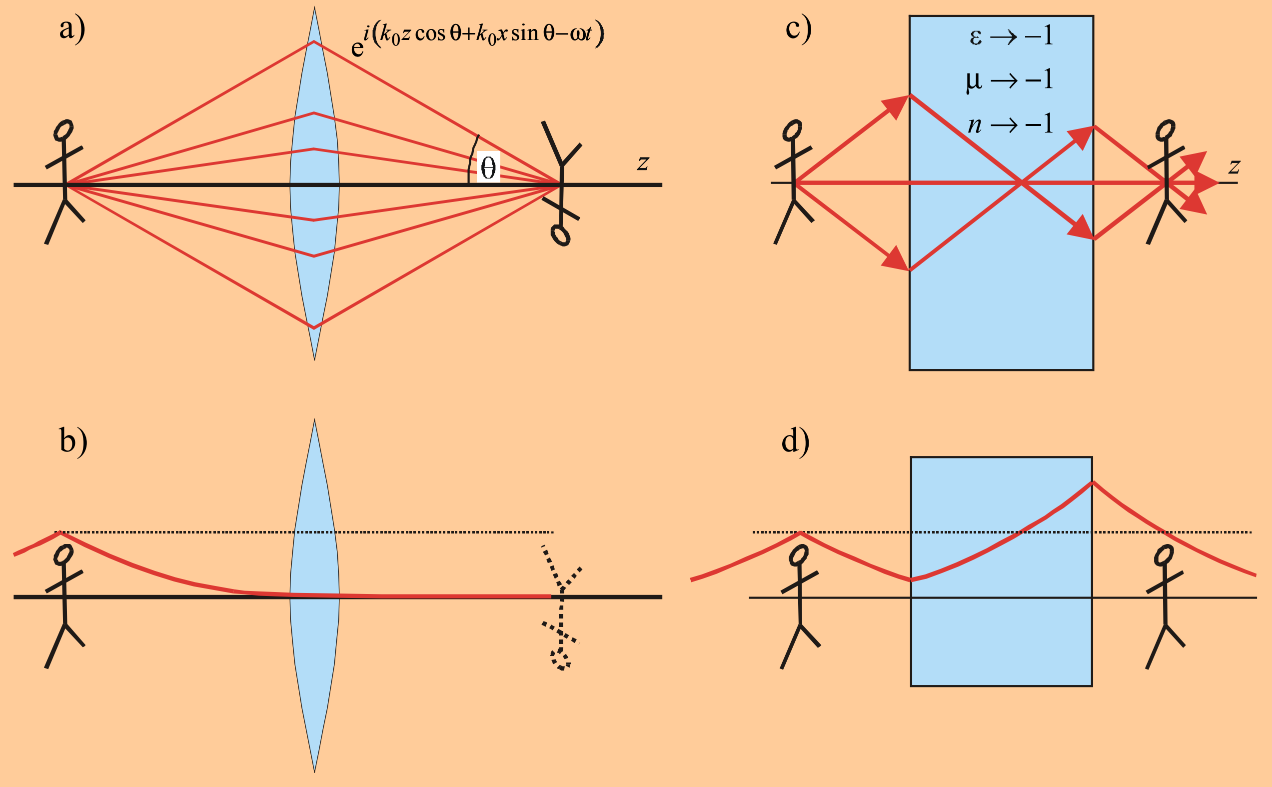

Figure 4. Limitation of resolution in lenses. a) Conventional lenses need a wide aperture for good resolution but even so are limited in resolution by the wavelength employed. b) The missing components of the image are contained in the near field which decays exponentially and makes negligible contribution to the image. c) A new lens made from a slab of negative material not only brings rays to a focus but has the capacity d) to amplify the near field so that it contributes to the image thus removing the wavelength limitation. However the resonant nature of the amplification places sever demands on materials: they must be very low loss.

curvature of the lens by $f = R/(n-1)$. The denominator in the focal length formula implies an inherent distinction between positive and negative index lenses, based on the fact that an $n=+1$ material does not refract electromagnetic fields while an $n=-1$ material does. The result is that negative index lenses can be more compact with a host of other benefits, as shown in figure 3.

To make a conventional lens with the best possible resolution a wide aperture is sought. Each ray emanating from an object, as shown in Figure 4a), has wave vector components along the axis of the lens, $k_z = k_0 \cos\theta$, and perpendicular to the axis, $k_x = k_0 \sin\theta$. The former component is responsible for transporting the light from object to image and the latter represents a Fourier component of the image for resolution: the larger we can make k_x, the better. Naturally the best that can be achieved is k_0 and hence the limit to resolution of ,

$$\Delta \approx \pi/k_0 = \lambda/2$$

where λ is the wavelength. This restriction is a huge problem in many areas of optics. Wavelength limits the feature size achieved in computer chips, and the storage capacity of DVDs. Even a modest relaxation of the wavelength limitation would be of great value.

In contrast to the image, there is no limit to the electromagnetic details contained in the object but unfortunately not all of them make it across the lens to the image. The problem lies with the z component of the wave vector which we can write,

$$k_z = \sqrt{k_0^2 - k_x^2}$$

Evidently for large values of k_x, corresponding to fine details in the object, k_z is imaginary and the waves acquire an evanescent character. By the time they reach the image they have negligible amplitude, figure 4b), and for this reason are commonly referred to as 'the near field', and the propagating rays as 'the far field'.

If by some magic we could amplify the near fields we could in principle recoup their contribution, but the amplification would have to be of just the right amount and possibly very strong for the most localized components. This is a tall order but by a remarkable chance the new negative slab lens achieves this feat[15].

In figure 4c) we see rays contributing to the image for the negative slab. Just as for the conventional lens, the rays only contribute details greater that about half a

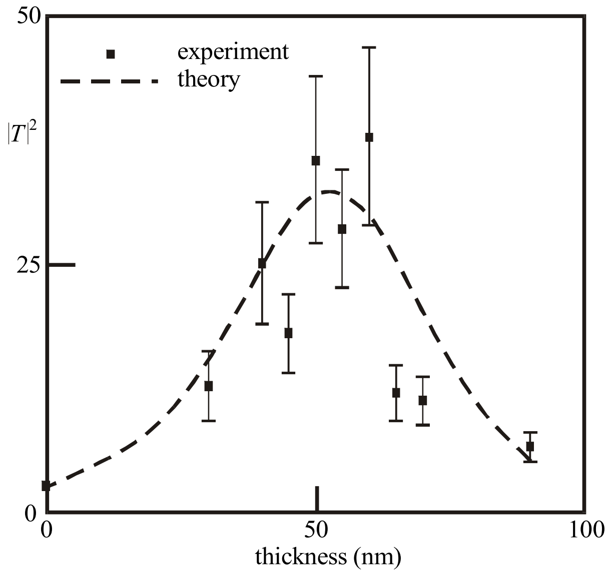

Figure 5. Amplifying the near field: near fields incident on thin films of silver are transmitted to the far side with amplification, at least up to a critical thickness. For thicker films losses dominate and spoil the resonant effect. Data taken form a paper by Fang et al.[11].

wavelength in diameter. In contrast the behavior of the near field is remarkably different as shown in figure 4d). It has the capacity to excite short wavelength resonances of the negative surface which are akin to the surface plasmons familiar on the surfaces of metals such as silver. Interaction with the plasmon like excitation kicks the decaying wave into the corresponding growing wave and the negative medium amplifies the wave, compensating for the decay that occurred in an equal thickness of vacuum. the resonances have a finite width and therefore this super lensing effect is a narrow band phenomenon: the requirement of $\varepsilon \to -1$, $\mu \to -1$ can be met only at one frequency because negative media are necessarily dispersive.

For a conventional lens resolution is limited by the aperture. Our new lens based on negative materials will also in practice have limitations, in this case chiefly due to losses. Any real material will always have small positive imaginary components to ε and μ which represent resistive losses in the system and damp the resonances responsible for amplifying the near fields. The sharpest resonances are the first to be killed by the losses and so with increasing loss the resolution is rapidly degraded. Fang *et. al.*[16] have explored these effects exploiting the fact that for very small systems much smaller than the free space wavelength we can concentrate on either the electric or magnetic fields. Silver has a negative real part to ε and therefore a thin film should behave like our negative slab and amplify the near field. They experimented on several

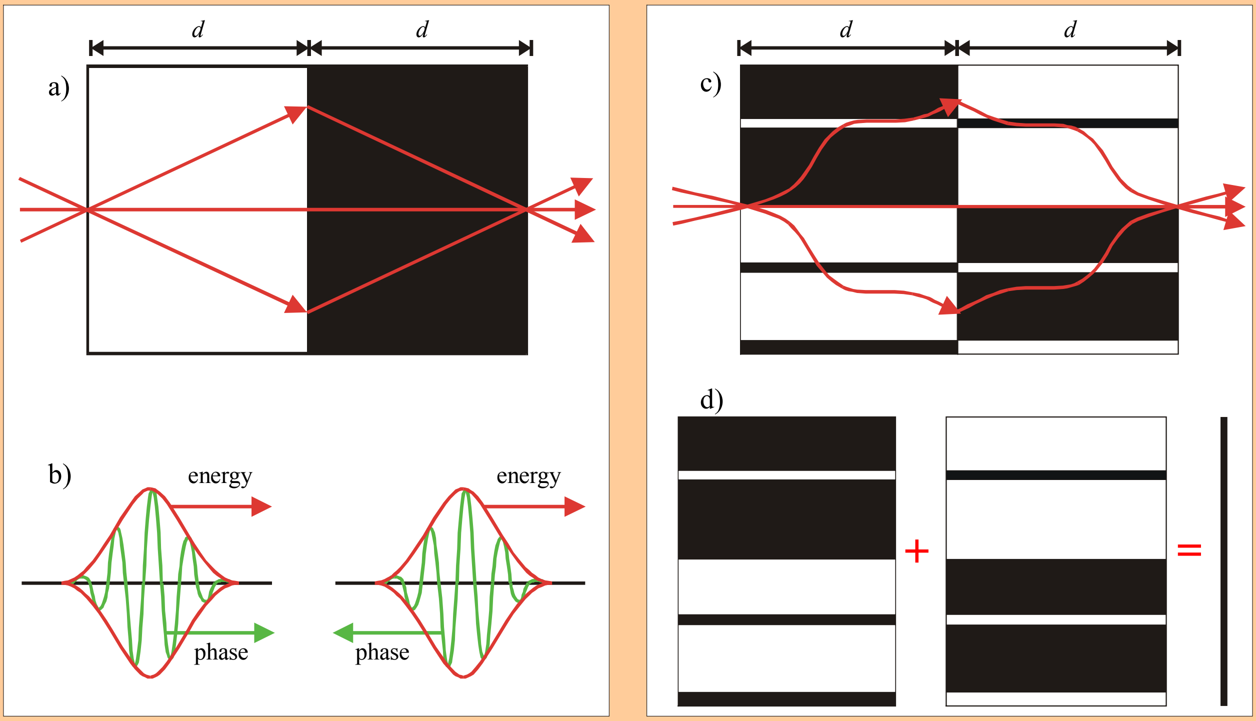

Figure 6. Generalizing the perfect lens: a) an $n = -1$ slab draws light to a perfect focus; b) shows how the focus is achieved by the negative slab 'unwinding' or negating the phase acquired in passing through free space; c) focusing can occur through two more complex objects provided that one is the inverse mirror image of the other; d) a graphical statement of the optical cancellation mirror antisymmetric regions of space optically annihilate one another. A negative medium is in effect a piece of optical antimatter.

films of different thickness but each time selecting the same k_x. Their results are seen in figure 5: the film is clearly amplifying waves up too a critical thickness, when losses intervene and the amplification process collapses. nevertheless considerable amplification is possible and this leads us to be optimistic that some limited sub wavelength focusing can be achieved with silver films.

Negative Refraction: Negative Space

We have seen that a slab of negative material with $\varepsilon = \mu = -1$ acts like a lens: objects on one side are brought to a focus on the other side. Figures 6a) and b) show that as the waves enter the negative medium, their phase is wound backwards as they progress. Overall the slab undoes the effect of an equal thickness of vacuum. Similarly decaying waves have their amplitude restored by passing through the slab. This suggests another view of the focusing action, that of the slab annihilating an equal thickness of vacuum. Negative media behave like optical antimatter.

In fact the result is more general than this. It has been shown[17] that two slabs of material optically annihilate if one is the negative mirror image of the other. Suppose that they meet in the plane $z = 0$, then at equal and opposite distances from this plane:

$$\varepsilon(x, y, z) = -\varepsilon(x, y, -z)$$

$$\mu(x, y, z) = -\mu(x, y, -z)$$

In figures 6c) and d) there is an illustration of what this might mean. The two media have varying refractive indices and so in general light does not follow a straight line. Nevertheless in each medium complementary paths are traced such that the overall phase acquired in the first medium is cancelled by the contribution from the second. Likewise if the waves have a decaying nature, decay in one half would be followed by amplification in the second.

This may seem straightforward but some configurations have surprises. Consider figure 7. In the top panel the two halves are inverse mirror images as required by the theorem, and therefore we expect that incident waves are transmitted without attenuation and without reflection. Yet a ray tracing exercise holds a surprise. Ray 2 in the figure hits the negative sphere and is twice refracted to be ejected from the system rather than transmitted. The rays are not supportive of our theorem! Further investigation shows that the sphere is capable of trapping rays in closed orbits, shown by dotted lines in the center of the figure. This is the signature of a resonance and a clue as to how the paradox is resolved. A full solution of Maxwell's equations shows that when the incident light is first switched on the ray predictions are initially obeyed. With time some of the incident energy will feed into the resonant state in the middle of the system which in turn will leak energy into a transmitted wave, and a contribution to the reflected wave which cancels with the original reflection. As always in negative

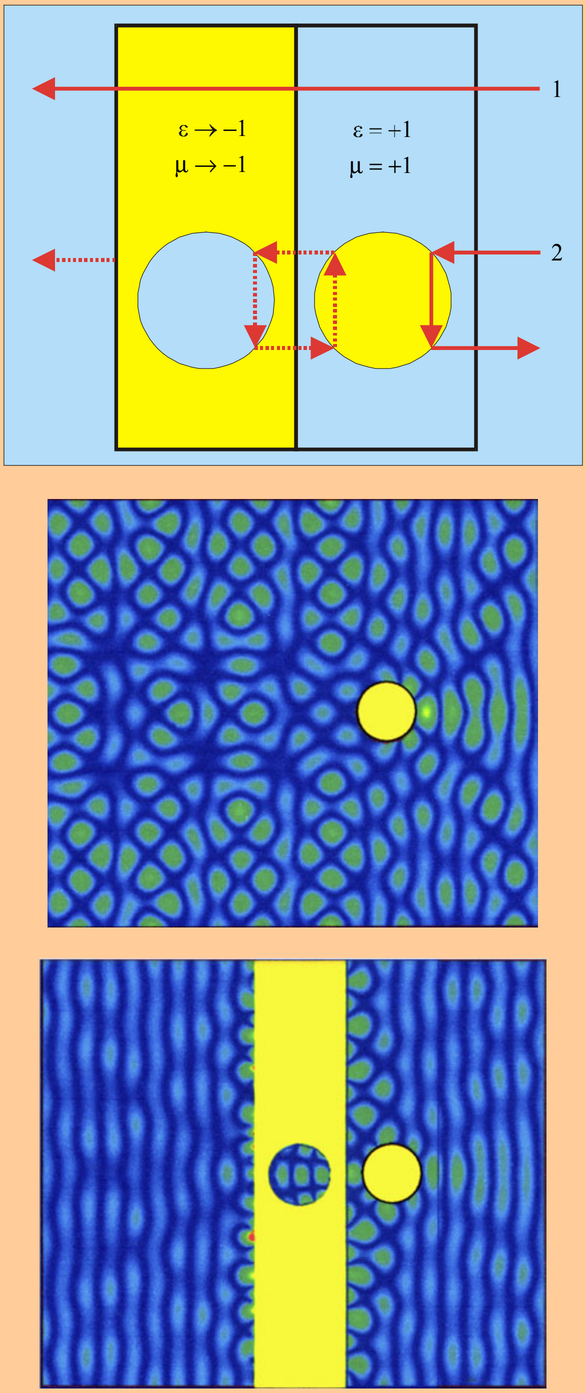

Figure 7. An optical paradox. In the top figure ray tracing predicts that some rays will be rejected from this system even though the mirror theorem predicts that all waves should be transmitted. The middle figure gives the solution of Maxwell's equations for a single negative cylinder $(n = -1)$ and the lower figure the solution when the complementary layer is added. In the latter figure within the accuracy of the calculation all scattering is removed. Note the accumulation of amplitude around the sphere indicating a resonant contribution to transmission.

media, resonant states play a central role in their properties.

The two lower panels in the figure show the resulting equilibrium solution first with only the negative sphere indicating that there is strong scattering, and then with the mirror symmetric layer included which within the accuracy of our calculation removes the reflected contributions and the spurious forward scattering to leave transmission unhindered as predicted.

An interesting question arises if there is absorption in the system represented by positive imaginary parts of either or both of ε and μ . Conditions for the theorem may still be satisfied but require that for every instance of a positive part to ε,μ there is a mirror antisymmetric negative ε,μ somewhere else in the system. This implies that parts of the system must exhibit gain. Loss can only be compensated by active amplification.

Conclusions

Negative refraction is a subject with constant capacity for surprise: innocent assumptions lead to unexpected and sometimes profound consequences. This has generated great enthusiasm but also controversy yet even the controversies have had the positive effect that key concepts have been critically scrutinized in the past 18 months. Finally in the past year experimental data have been produced which validate the concepts. As a result we have a firm foundation on which to build. Many groups are already moving forward with applications. Naturally the microwave area has been most productive as the metamaterials required are easier to fabricate. We have given an illustration of a microwave lens, but novel waveguides and other devices are under consideration.

One of the most exciting possibilities is imaging beyond the wavelength limit. Practical applications will require low loss materials, a great challenge to the designers of new metamaterials. Proposals to employ thin silver films as lenses are under investigation in several laboratories. Nor are the challenges purely experimental: we are not yet done with theory since the assumption of negative refraction has many ramifications still being explored and which are sure to cast more light on this strange but fascinating subject. Not surprisingly many are joining the field and 2003 saw over 200 papers published on negative refraction. We expect even more in 2004!

Further reading can be found in a special edition of Optics Express[18] and in the article by McCall *et al*[19] .

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