Nonlinear propagation of electromagnetic waves in negative-refraction-index composite materials

I. Kourakis[1,2,*] and P. K. Shukla[1,†]

[1]*Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

[2]*Max-Planck-Institut für Extraterrestrische Physik, Giessenbachstrasse, D-85740 Garching, Germany*

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We investigate the nonlinear propagation of electromagnetic waves in left-handed materials. For this purpose, we consider a set of coupled nonlinear Schrödinger (CNLS) equations, which govern the dynamics of coupled electric and magnetic field envelopes. The CNLS equations are used to obtain a nonlinear dispersion, which depicts the modulational stability profile of the coupled plane-wave solutions in left-handed materials. An exact (in)stability criterion for modulational interactions is derived, and analytical expressions for the instability growth rate are obtained.

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I. INTRODUCTION

An increasing amount of research work has recently been focusing on *left-handed* (or *negative index*) materials (LHM), i.e., artificially produced materials which are characterized by a negative value of both the dielectric permittivity ϵ and magnetic permeability μ, in addition to a negative value of the refraction index $n=-\sqrt{\epsilon\mu}$. Although the original theoretical concept of LHM dates back in the seminal work of Veselago [1], the absolute absence of naturally occurring materials of this kind has restricted attention to Veselago's speculations until very recently. Since a number of pioneering theoretical studies [2–4] suggested how these peculiar properties could be realized in purpose-designed and built materials, and experiments subsequently confirmed those predictions [5–8], the field of LHM has received a considerable boost in the last half decade, overcoming an initial controversial phase of lack of theoretical consensus and acceptance (read, e.g., [9] for a recent review). Interestingly, a number of alternative theoretical schemes bearing left-handed electromagnetic behavior were recently proposed, including RLC transmission lines [10], photonic circuits [11], and other nanostructures [12]. Furthermore, a number of applications (e.g., in optics [13]) were suggested to exploit the singular physical properties of LHM (beam refocusing, inversion of Snell's law and of the Doppler shift effect, backward Cerenkov radiation, etc). It may be noted that one is interested in media characterized by *both* ϵ and μ being negative, since the mixed case (negative-positive) has been shown in Ref. [5] to bear a reduced transmittivity (i.e., the medium is *opaque* to electromagnetic waves).

Naturally, the theory of the propagation of electromagnetic (EM) waves in linear LHM [14] was recently extended to account for nonlinear (i.e., field-amplitude-dependent) material properties [15,16]. *Ab initio* calculations of the nonlinear dielectric and magnetic properties of split-ring resonator (SRR) lattice structures showed that magnetic nonlinearity, in principle, *dominates* in LH composite materials [15]. Taking this fact into account, the dynamics of the electric and magnetic field envelope of an EM wave propagating in a LH medium was recently related to the nonlinear amplitude modulation formalism [17] by Lazarides *et al.*, who showed that modulated EM wave propagation is governed by a pair of coupled nonlinear Schrödinger-type equations (CNLS). Such nonlinear equations generally describe the dynamics of a slowly varying envelope, which confines (*modulates*) the fast carrier wave [18–20].

In this paper, we present an investigation of the nonlinear stability of electromagnetic waves in a negative-refractive-index medium or LHM. For this purpose, we use the CNLS equations to obtain a nonlinear dispersion relation. The latter is analyzed both analytically and numerically to demonstrate the nonlinear stability or instability of a modulated electromagnetic wave packet in left-hand composite materials.

II. NONLINEAR DESCRIPTION OF EM WAVE PROPAGATION IN LHM

The dielectric and magnetic behaviors of negative index materials or LHM are characterized by both frequency *dispersion* (as physically imposed [1]) and *nonlinearity* [15,16]. In the following, we shall briefly review the existing theories modeling these mechanisms, in order to set the theoretical background of the modulation stability analysis that will follow.

A. Nonlinear LHM properties

The dielectric and magnetic response of a nonlinear material is formally characterized by an electric flux density $\mathbf{D}$ and a magnetic induction $\mathbf{B}$, which depend on the electric and magnetic field intensities $\mathbf{E}$ and $\mathbf{H}$ as $\mathbf{D}=\epsilon_{eff}\mathbf{E}=\epsilon\mathbf{E}+\hat{\mathbf{P}}$ and $\mathbf{B}=\mu_{eff}\mathbf{H}=\mu\mathbf{H}+\hat{\mathbf{M}}$, where ϵ and μ denote the medium (linear) *dielectric permittivity* and *magnetic permeability*, respectively, while $\hat{\mathbf{P}}=\epsilon_{\mathrm{NL}}\mathbf{E}$ and $\hat{\mathbf{M}}=\mu_{\mathrm{NL}}\mathbf{H}$ express the *nonlinear* contributions to the medium polarization and magnetization. We note that, throughout this text, our notations incorporate the vacuum dielectric permittivity ϵ_0 and magnetic permeability $\mu_0=1/c^2\epsilon_0$ into ϵ_{eff} and μ_{eff} (contrary to a

*Electronic address: ioannis@tp4.rub.de; www.tp4.rub.de/~ioannis.

†Electronic address: ps@tp4.rub.de; www.tp4.rub.de/~ps.

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widely used notation, according to which $\mathbf{B}=\mu_{eff}\mu_0\mathbf{H}$ and $\mathbf{D}=\epsilon_{eff}\epsilon_0\mathbf{E}$).

In specific, if one neglects energy dissipation (loss), the dielectric response of LH composite materials (SRR lattices here) is given by the nonlinear and dispersive (frequency-dependent) expression [15]

$$\epsilon_{eff}=\epsilon_0\left(1-\frac{\omega_p^2}{\omega^2}\right)+\epsilon_{\rm NL}(|E|^2) \equiv \epsilon+\epsilon_{\rm NL}(|E|^2) \equiv \epsilon_D(|E|^2)-\frac{\omega_p^2}{\omega^2}, \quad (1)$$

where the effective cutoff ("plasma") frequency ω_p is related to the geometrical features of the SRR lattice [2,15], ω is the propagating mode frequency, and ϵ_D is related to the (nonlinear) dielectric response. We observe that ϵ_0 sets the asymptotic limit of ϵ for $\omega\to\infty$. The possibility for negative permittivity arises from the frequency dependence for $\omega<\omega_p$.

The magnetic response of LH (in fact of SRR lattice-based) materials was studied by Pendry *et al.* [3], whose result was generalized by Zharov *et al.* (also see [16]) to the nonlinear expression

$$\mu_{eff}=\mu_0\left(1+\frac{F\omega^2}{\omega_{0,\rm NL}^2(|H|^2)-\omega^2}\right) =\mu_0\left(1+\frac{F\omega^2}{\omega_0^2-\omega^2}\right)+\mu_{\rm NL}(|H|^2)\equiv\mu+\mu_{\rm NL}(|H|^2), \quad (2)$$

where the (linear) resonant SRR frequency ω_0 and the factor F (denoting the single ring to unit cell area ratio; $0<F\ll 1$, ideally) are related to the intrinsic lattice structure [3]. Again, we have neglected dissipation here. The (linear) permeability μ attains negative values for $\omega_0<\omega<\omega_0/\sqrt{1-F}$, while the (complete) effective permeability μ_{eff} yields an enriched behavior (cf. the discussion in Refs. [3,15]; see below). We note that $\mu\to\mu_0$ for $\omega\to 0$.

The nonlinear frequency $\omega_{0,\rm NL}\equiv X\omega_0$ is related to the mode and resonance frequencies, $\omega\equiv\Omega\omega_0$ and ω_0, via a complex expression, which is in turn related to the material's dielectric properties, viz., $|H|^2=f(X,\Omega;\epsilon_{\rm NL}(|E|^2))$ [15]. For a relatively small field, one may assume a Kerr dielectric material behavior, i.e., $\epsilon_{eff}\approx\epsilon+\alpha|E|^2$ (the nonlinearity parameter α is related to intrinsic material properties; here it is equal to αE_c^2 in Ref. [15,16]); a positive (negative) value of α denotes a focusing (defocusing) dielectric behavior. One then has [15–17]

$$|H|^2=\alpha A^2E_c^4\frac{(1-X^2)(X^2-\Omega^2)^2}{\Omega^2X^6}, \quad (3)$$

where E_c determines the characteristic dielectric nonlinearity scale (viz., $\alpha=\pm E_c^{-2}$) and the quantity A is related to the physical features of the material unit elements [15,16]. Note that this functional relation suggests a multivalued dependence of X, and consequently of μ_{eff}, on $|H|^2$ (cf. Fig. 2 in Ref. [15]).

For the sake of analytical tractability, one may consider a "Kerr-like" dependence, viz., $\mu_{eff}\approx\mu+\beta|H|^2$, where the parameter β is related to intrinsic material properties. Although, given the complexity of Eq. (3), it is not trivial to obtain an analytical expression for the phenomenological nonlinearity parameter β, this assumption seems to be justified (and may be numerically confirmed [17]) for sufficiently low values of the magnetic field intensity H; also cf. [15], wherein the negative (stable) curves in Figs. 2(a), 2(b), and 2(c), respectively, reflect the LH behaving cases ($\omega>\omega_0$ and $\alpha,\beta>0$), ($\omega<\omega_0$ and $\alpha,\beta>0$), and ($\omega>\omega_0$ and $\alpha,\beta<0$), here. We note that the case ($\omega<\omega_0,\alpha,\beta<0$) bears no left-handed behavior [as depicted in Fig. 2(d) in Ref. [15]].

We recall that the dispersive character of the medium is hidden in the frequency dependence of both ϵ and μ, and that, in fact, left-handed behavior is restricted within a certain range of frequency values. Thus, the above formulation formally applies to both the right-handed and left-handed behaving frequency ranges of the composite materials mentioned above. Nevertheless, rigorously speaking, the above description refers to low magnetic fields, as explained above. Finally, we note that the above relations are compatible with the causality requirements $d[\epsilon(\omega)\omega]/d\omega>1$ and $d[\mu(\omega)\omega]/d\omega>1$, as pointed out in Ref. [4].

B. EM wave modulation

Let us consider an EM plane wave propagating in a left-handed medium. The wave consists of an electric and a magnetic field(s) of intensities $\mathbf{E}$ and $\mathbf{H}$, respectively, representing transverse propagating oscillations in perpendicular directions. We recall that $\mathbf{E}\times\mathbf{H}$ determines the (Poynting) direction of energy flow, which coincides (is opposed to) the propagation direction, say along z, in right-handed (RH) [left-handed (LH)] materials [4]. The field vector magnitudes are $E(z,t)=\mathcal{E}(z,t)\exp[i(kz-\omega t)]$ and $H(z,t)=\mathcal{H}(z,t)\exp[i(kz-\omega t)]$, where ω, $k=2\pi/\lambda$ and λ here denote the cyclic frequency, the wave number, and the wavelength, respectively. The propagation of the EM wave considered is governed by Maxwell's laws. The nonlinear modulation of the wave amplitude(s) was recently shown [17] to be governed by the system of coupled nonlinear Schrödinger (CNLS) equations

$$i\frac{\partial\mathcal{E}}{\partial T}+P\frac{\partial^2\mathcal{E}}{\partial X^2}+Q_{11}|\mathcal{E}|^2\mathcal{E}+Q_{12}|\mathcal{H}|^2\mathcal{E}=0, \quad (4)$$

$$i\frac{\partial\mathcal{H}}{\partial T}+P\frac{\partial^2\mathcal{H}}{\partial X^2}+Q_{22}|\mathcal{H}|^2\mathcal{H}+Q_{21}|\mathcal{E}|^2\mathcal{H}=0. \quad (5)$$

The (slow) position and time variables are defined as $X=\delta(x-v_gt)$ and $T=\delta^2t$, where $\delta\ll 1$ is a small real parameter. The field envelopes $\mathcal{E}$ and $\mathcal{H}$ move at the group velocity, which is related to the wave vector $\mathbf{k}$ as by $\mathbf{v_g}=c^2\mathbf{k}/\omega$ [viz. $v_g=\omega'(k)=c^2k/\omega$]. The (common) group velocity dispersion coefficient is $P=\omega''(k)/2=(c^2-\omega'^2)/2\omega$; we note that $P>0$, since the condition $v_g<c$ is prescribed by causality in both

RH and LH materials [4]. The frequency ω is related to the wave number k via a dispersion relation [related to the perplex expression for $\epsilon_{eff}(\omega)$ [4]], which to lowest order reads $\omega = k/\sqrt{\epsilon\mu} \equiv ck$. The nonlinearity coefficients $Q_{11}=Q_{21} = \omega c^2 \alpha\mu/2 \equiv Q_1$ and $Q_{22}=Q_{12}=\omega c^2 \beta\epsilon/2 \equiv Q_2$ are related to the nonlinearity ("Kerr") parameters α and β (both assumed $\sim \delta^2$ here). Note the peculiar symmetry of the nonlinear part of Eqs. (4) and (5) (contrary to the "usual" case in the nonlinear optics, where the self-interaction nonlinearity and cross-coupling coefficients are, separately, equal to each other, viz., $Q_{11}=Q_{22}$ and $Q_{12}=Q_{21}$, instead).

III. PLANE-WAVE SOLUTIONS—MODULATIONAL STABILITY ANALYSIS

In search of a set of coupled solutions to the CNLS equations above, one may set $\mathcal{E}(X,T)=\rho_1 \exp(i\theta_1)$ and $\mathcal{H}(X,T) = \rho_2 \exp(i\theta_2)$, where $\rho_{1,2}$ and $\theta_{1,2}$ are real functions of $\{X,T\}$, to be determined. Substituting in Eqs. (4) and (5), one readily obtains

$$\rho_{i,T} + P(2\rho_{i,X}\theta_{i,X} + \rho_i \theta_{i,XX}) = 0 \tag{6}$$

and

$$\theta_{i,T} = P[\rho_{i,XX}/\rho_i - (\theta_{i,X})^2] + Q_1\rho_1^2 + Q_2\rho_2^2, \tag{7}$$

where i and $j \neq 1$ are either 1 or 2, and the subscripts X and T denote partial differentiation, viz. $f_X \equiv \partial f/\partial X$ and so forth. Taking $\rho_{1,2}$=const, we obtain a set of coupled monochromatic envelope (Stokes) wave solutions in the form

$$\{\mathcal{E}(X,T), \mathcal{H}(X,T)\} = \{\mathcal{E}_0, \mathcal{H}_0\} e^{i(Q_1|\mathcal{E}_0|^2 + Q_2|\mathcal{H}_0|^2)T}. \tag{8}$$

These solutions represent two copropagating modulated field envelopes, oscillating (slowly) at a frequency $\Omega = (Q_1|\mathcal{E}_0|^2 + Q_2|\mathcal{H}_0|^2)$ (which depends on the constant linear field wave amplitudes $\mathcal{E}_0$ and $\mathcal{H}_0$). Note that the phase ΩT is common, due to the symmetry of the CNLS equations above.

In order to study the stability of the above monochromatic solution, we set $\mathcal{E}_0 \to \mathcal{E}_0 + \xi\mathcal{E}_1(X,T)$ and $\mathcal{H}_0 \to \mathcal{H}_0 + \xi\mathcal{H}_1(X,T)$, where the small ($\xi \ll 1$) perturbations $\mathcal{E}_1$ and $\mathcal{H}_1$ are *complex* functions of $\{X,T\}$. Isolating terms in ξ, we obtain

$$i\mathcal{E}_{1,T} + P\mathcal{E}_{1,XX} + Q_1(\mathcal{E}_1 + \mathcal{E}_1^*)\mathcal{E}_0^2 + Q_2(\mathcal{H}_1 + \mathcal{H}_1^*)\mathcal{E}_0\mathcal{H}_0 = 0,$$

along with the analogous equation for $\mathcal{H}_1$ (obtained upon the permutation $\mathcal{E}_1 \leftrightarrow \mathcal{H}_1$ and $Q_1 \leftrightarrow Q_2$). Separating real and imaginary parts, and assuming a perturbation wave number $\tilde{k}$ and frequency $\tilde{\omega}$, we obtain

$$[-\tilde{\omega}^2 + P\tilde{k}^2(P\tilde{k}^2 - 2Q_1\mathcal{E}_0^2)]\tilde{a}_1 - 2PQ_2\mathcal{E}_0\mathcal{H}_0\tilde{k}^2\tilde{a}_2 = 0 \tag{9}$$

and

$$-2PQ_1\mathcal{E}_0\mathcal{H}_0\tilde{k}^2\tilde{a}_1 + [-\tilde{\omega}^2 + P\tilde{k}^2(P\tilde{k}^2 - 2Q_2\mathcal{H}_0^2)]\tilde{a}_2 = 0. \tag{10}$$

This system is tantamount, formally, to the eigenvalue problem $(\mathbf{M} - \tilde{\omega}^2\mathbf{I})\tilde{\mathbf{a}} = \mathbf{0}$, where the elements of the vector $\tilde{\mathbf{a}} = (a_1, a_2)^T$ are the perturbation's amplitudes, $\mathbf{I}$ is the unit matrix ($I_{ij}=\delta_{ij}$, for $i,j=1,2$), and the elements of the matrix $\mathbf{M}$ are $M_{11}=P\tilde{k}^2(P\tilde{k}^2-2Q_1\mathcal{E}_0^2)$, $M_{22}=P\tilde{k}^2(P\tilde{k}^2-2Q_2\mathcal{H}_0^2)$, $M_{12} = -2PQ_2\mathcal{E}_0\mathcal{H}_0\tilde{k}^2$, and $M_{21}=-2PQ_1\mathcal{E}_0\mathcal{H}_0\tilde{k}^2$. The condition for the existence of eigenvalues, viz., $\det(\mathbf{M}-\tilde{\omega}^2\mathbf{I})=0$, provides the bi-quadratic polynomial equation

$$\tilde{\omega}^4 - T\tilde{\omega}^2 + D = 0, \tag{11}$$

where $T=M_{11}+M_{22} \equiv \mathrm{Tr}\,\mathbf{M}$ and $D=M_{11}M_{22}-M_{12}M_{21} \equiv \det\mathbf{M}$ denote the trace and the determinant, respectively, of the matrix $\mathbf{M}$. Evaluating T and D, we find $T=2P^2\tilde{k}^2(\tilde{k}^2 - K)$ and $D=P^4\tilde{k}^6(\tilde{k}^2-2K)$, where we defined the quantity $K=(Q_1|\mathcal{E}_0|^2+Q_2|\mathcal{H}_0|^2)/P$. The discriminant quantity $T^2-4D = 4P^4\tilde{k}^4K^2 \geq 0$ turns out to be non-negative, so Eq. (11) yields two real solutions for $\tilde{\omega}^2$, viz.,

$$\tilde{\omega}_\pm^2 = \frac{1}{2}(T \pm \sqrt{T^2 - 4D}), \tag{12}$$

or, explicitly,

$$\tilde{\omega}_+^2 = P^2\tilde{k}^4, \quad \tilde{\omega}_-^2 = P^2\tilde{k}^2(\tilde{k}^2 - 2K). \tag{13}$$

We immediately see that $\omega_+ = \pm P\tilde{k}^2 \in \mathbb{R}$, while, on the other hand, the sign of ω_-^2 (hence the existence or not of a nonzero imaginary part of ω_-) depends on the quantity $\tilde{k}^2 - 2K$. In specific, if the following criterion is met:

$$\tilde{k}^2 - \frac{2}{P}(Q_1|\mathcal{E}_0|^2 + Q_2|\mathcal{H}_0|^2) > 0, \tag{14}$$

or, recalling the definitions of $Q_{1,2}$ above,

$$\tilde{k}^2 - \frac{\omega}{P}\left(\frac{\alpha}{\epsilon}|\mathcal{E}_0|^2 + \frac{\beta}{\mu}|\mathcal{H}_0|^2\right) \equiv \tilde{k}^2 - \frac{\omega}{P}K' > 0, \tag{15}$$

then the EM wave will be modulationally stable. Since $P>0$, we see that the EM stability profile will essentially depend on the quantity K' (within parentheses). In the existing description of "ordinary" RH materials, one has $\mu, \epsilon > 0$, so (for $\beta=0$, say, i.e., for a linear magnetic response) a modulational instability may or may not occur, depending on the focusing or defocusing dielectric property of the medium (i.e., on the sign of α). However, in LHM, both μ and ϵ are negative, while α and β depend on the medium's structure. Clearly, the EM wave will be *stable* if the quantity

$$K' = \frac{\alpha}{\epsilon}|\mathcal{E}_0|^2 + \frac{\beta}{\mu}|\mathcal{H}_0|^2 \tag{16}$$

is *negative* [and thus Eq. (15) is satisfied $\forall k$]. If, on the other hand, K' is *positive*, the EM wave will be unstable to external perturbations with a wave number $\tilde{k}$ lower than $\tilde{k}_{cr} \equiv \sqrt{2K} = \sqrt{\omega K'/P}$ (see definitions above). The growth rate $\sigma = i\sqrt{-\tilde{\omega}_-^2}$ of the instability then attains its maximum value $\sigma_{\max} = PK = \omega K'/2 = \omega(\alpha|\mathcal{E}_0|^2/\epsilon + \beta|\mathcal{H}_0|^2/\mu)/2$ at $\tilde{k} = \sqrt{K} = \tilde{k}_{cr}/\sqrt{2}$.

In order to study the numerical behavior of the (lower branch of the) perturbation dispersion relation (13), one may

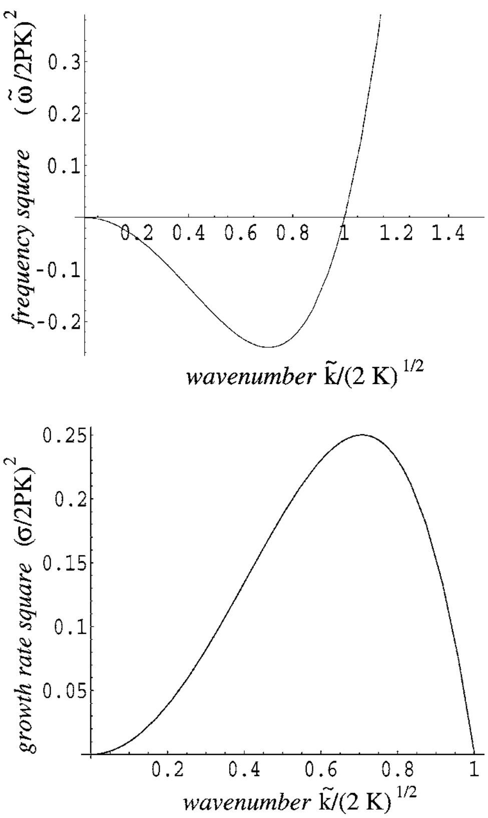

FIG. 1. (a) The square of the perturbation frequency $\tilde{\omega}$ (scaled by $2KP=\omega K'$) is depicted vs the perturbation wave number $\tilde{k}$ (scaled by $\sqrt{2K}$), as derived from the relation (13) for $K>0$ (unstable case). (b) The square of the instability growth rate σ (scaled by $2KP=\omega K'$) is depicted vs the perturbation wave number $\tilde{k}$ (scaled by $\sqrt{2K}$).

express it as $\hat{\omega}_-^2=\hat{k}^2(\hat{k}^2-1)\equiv\Omega(\hat{k})$, where $\hat{\omega}_-^2=\omega_-^2/(\omega^2K'^2)$ and $\hat{k}^2=\tilde{k}^2/2K=P\tilde{k}^2\omega K'$ (we assume $K'>0$ here, to study instability; the opposite case $K'<0$ would be stable). The function $\Omega(\hat{k})$ is depicted in Fig. 1(a); for $\hat{k}<1$, it bears negative values, implying instability, viz., $\hat{\omega}_-^2\equiv-\hat{\sigma}^2<0$. The associated growth rate attains its maximum value $\hat{\sigma}_{\max}=1/2$ at $\hat{k}=1/\sqrt{2}$; see Fig. 1(b). On the other hand, if $\hat{k}>1$, then $\hat{\omega}^2>0$ (hence $\hat{\omega}$ is real) and the EM wave will be stable. The field amplitude(s) will then oscillate around the stationary (constant amplitude) state, but will otherwise retain their stability against the external perturbation. We see that the stable and unstable wave number ranges are separated by the critical value $\hat{k}_{cr}=1$, which corresponds to a perturbation wave number $\tilde{k}_{cr}=(2/P)^{1/2}(Q_1|\mathcal{E}_0|^2+Q_2|\mathcal{H}_0|^2)^{1/2}$.

The stability criterion $K'<0$, where K' is defined in Eq. (16), may be investigated with respect to real material values, provided by experiments. It may be instructive to refer to the cases depicted in Fig. 2 in Ref. [15]. For instance, we see that (the negative branch of) Fig. 2(a) therein, referring to the case (ϵ, $\mu<0$ and α, $\beta>0$), corresponds to a modulationally stable EM wave propagation (since $K'<0$). On the other hand, (the stable negative branch of) Fig. 2(c) in Ref. [15], which refers to the case ($\epsilon,\mu<0$ and $\alpha,\beta<0$), corresponds to *unstable* EM waves (since $K'<0$). Finally, EM waves propagating in the medium depicted in Fig. 2(d) (in Ref. [15]), which refers to the (opaque, see above) case ($\epsilon<0<\mu$, assuming $\omega<\omega_p$, and $\alpha,\beta<0$), may be potentially *unstable*, depending on the relative magnitude of the field amplitudes E_0 and H_0.

It is interesting to note that the quantity whose sign determines the stability profile of the EM wave, according to our analysis, is essentially

$$K' \approx \epsilon_{eff}/\epsilon+\mu_{eff}/\mu-2 \qquad (17)$$

[to lowest order in nonlinearity, i.e., $\mathcal{O}(|E|^2,|H|^2)$]. Quite expectedly, from a physical point of view, the nonlinear profile of EM waves in LH media is thus exactly related to the intrinsic nonlinear properties of the response of the media. The well known focusing/defocusing nonlinearity criterion, related to the Kerr property of a medium, is thus generalized to account for the intrinsically nonlinear magnetization properties of a left-handed material.

IV. CONCLUSIONS

We have investigated, from first principles, the conditions for the occurrence of the modulational instability in left-handed materials. Relying on a set of coupled nonlinear Schrödinger-type equations for the electric and magnetic field envelopes, we have shown that an electromagnetic wave consisting of two modulated field waves may become modulationally unstable. An exact criterion for (in)stability has been derived and analytical expressions for the instability growth rate have been obtained. The modulational instability is a well known mechanism of energy localization in nonlinear dispersive media, associated with the formation of propagating localized excitations. The present investigation aims at contributing to this new direction of thought, with respect to novel LH materials.

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